Undergraduate Project (UGP-1) Final Report

Hughe’s Pedestrian Flows

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Introduction to Crowd Dynamics

What is it?

Crowd dynamics is the study of how large groups of people move and interact, focusing on their collective behavior in different environments.

Why is it?

Studying crowds and their behavior will help us to predicting it in a given environment. This will allow us to predict the crowd movement.

Being able to predict the crowd will allow us to manage it efficiently.

How is it done?

Crowd simulation is done my coming up with models with appropriate assumptions and solving them numerically.
(just like another field of science)

There are two ways to think about crowds while modelling them.
We could either choose to model each pedestrian individually, which is called **Microscopic Model**, or we could model crowd as continuum, which is **Macroscopic Models.**

I, for the purpose of my project, have choose to use Macroscopic Models.

And the model chosen is the Hughe's continuum Model

Content

This is going to be the flow of presentation:

Hypothesis

↓

Governing Equations and Boundary Conditions

↑

Numeric Methods

↓

Results

1.Hughe’s Flow (Assumptions)

Assumption 1:

Speed of the pedestrian is determined by the local density around them.

Higher the density at a given point, lower will be the velocity at that point and vice versa.

Also defined is **Jammed Density** at which speed of the pedestrians is zero. And **Maximum velocity,** velocity when density around the pedestrian is equal to zero.

The relation chosen for this project is **Green Sheild’s Relation** between speed (f(rho)) and density (rho).

$$f(\rho) = V_{Max}\left(1 - \frac{\rho}{\rho_{Max}}\right)$$

Assumption 2:

A potential Field exist such that pedestrian move right angle to the line of constant potential to minimize it.

Potential Field could be assumed to be the **travel time**. So potential is a **Scalar Field** which could be defined at each point in the domain as a scalar, and it tells how much time a pedestrian at that point, will take to reach the exit.

Assuming **phi** to be potential and **u** as X-component and **v** as y component of velocity

$$u = f(\rho)\,\frac{\left(-\frac{\partial \phi}{\partial x}\right)}{\|\nabla \phi\|} \qquad v = f(\rho)\,\frac{\left(-\frac{\partial \phi}{\partial y}\right)}{\|\nabla \phi\|}$$

1.Hughe’s Flow (Assumptions)

Assumption 3:

The rate of increase in minimum travel time with distance is inversely proportional to walking speed.

This means at places where density is high (lower speed) the change in travel time per unit distance is higher than places where the density is low (Higher speed).

This is a differential equation and is known as **Eikonal Equation** (Where f represents speed and phi is potential) **:**

$$f(\rho) = \frac{1}{\|\nabla \phi\|} = \frac{1}{\left[\left(\frac{\partial \phi}{\partial x}\right)^2 + \left(\frac{\partial \phi}{\partial y}\right)^2\right]^{1/2}}$$

Assumption 4:

The number of pedestrians are conserved.

This comes directly from fluid mechanics, and it means that pedestrian could not be created, or destroyed.

Advection Equation:

Rho represents density and **u,v** represents component of velocity

$$\frac{\partial \rho}{\partial t} + \nabla.(\rho \vec{u}) = 0$$

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) = 0$$

2. Governing Equations

Advection Equation

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \bar{u}) = 0$$

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) = 0$$

Eikonal Equation

$$f(\rho) = \frac{1}{\|\nabla \phi\|} = \frac{1}{\left[\left(\frac{\partial \phi}{\partial x}\right)^2 + \left(\frac{\partial \phi}{\partial y}\right)^2\right]^{1/2}}$$

Speed-Density Relation

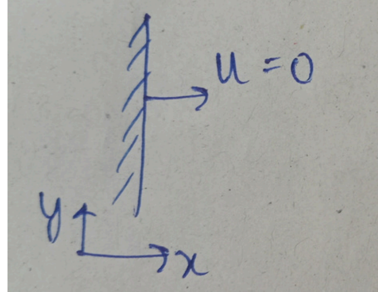
$$f(\rho) = V_{Max}\left(1 - \frac{\rho}{\rho_{Max}}\right)$$

Velocity-Potential Relation

$$u = f(\rho)\,\frac{\left(-\frac{\partial \phi}{\partial x}\right)}{\|\nabla \phi\|} \qquad v = f(\rho)\,\frac{\left(-\frac{\partial \phi}{\partial y}\right)}{\|\nabla \phi\|}$$

Boundary Conditions :

At walls the normal component of velocity must be zero (No penetration condition):



$$u = f(\rho)\,\frac{-\frac{\partial \phi}{\partial x}}{\|\nabla \phi\|} = 0$$

$$\Rightarrow -\frac{\partial \phi}{\partial x}\bigg|_{wall} = 0$$

At the wall the density is equal to be zero.

$$\rho\big|_{wall} = 0$$

Exit time at the exit will be zero.

$$\phi\big|_{exit} = 0$$

At inflow condition will depend on the problem and no condition on outflow is considered.

Numeric Methods

In crowd Simulation there could be various ways to simulate the governing equations, and **I have chosen Finite Difference Approach to do so, in my project.**

Advection Equation

The Finite Difference scheme used for solving advection equation is **Lax Wendroff Scheme**

Lax wendroff scheme:

$$\rho^{n+1/2}_{j+1/2} = \frac{1}{2}\left(\rho^n_{j+1} + \rho^n_j\right) - \frac{\Delta t}{2\Delta x}\left(F^n_{j+1} - F^n_j\right)$$

$$\rho^{n+1/2}_{j-1/2} = \frac{1}{2}\left(\rho^n_j + \rho^n_{j-1}\right) - \frac{\Delta t}{2\Delta x}\left(F^n_j - F^n_{j-1}\right)$$

$$\rho^{n+1}_j = \rho^n_j - \left(\frac{\Delta t}{\Delta x}\right)\left(F^{n+1/2}_{j+1/2} - F^{n+1/2}_{j-1/2}\right)$$

~~And~~ where, $F^n_j = f(\rho^n_j)\cdot\rho^n_j$

$= (1-\rho^n_j)\cdot\rho^n_j$

Eikonal Equation

The Finite Difference scheme used for solving advection equation is **Fast Sweep Scheme**

Given equation: $|\nabla\phi(x)| = f(x)$ on Ω

$\phi(x) = 0$ on Γ_{exit}

$\frac{\partial\phi}{\partial n} = 0$ on Γ_{wall}

$$\phi^x_{Min} = \min\left(\phi^{(n)}_{i-1,j}, \phi^{(n)}_{i+1,j}\right)$$

$$\phi^y_{Min} = \min\left(\phi^{(n)}_{i,j-1}, \phi^{(n)}_{i,j+1}\right)$$

$$\phi^{(n+1)}_{ij} = \begin{cases} \min\left(\phi^x_{Min}, \phi^y_{Min}\right) + f_{ij}h \; ; & |\phi^x_{Min} - \phi^y_{Min}| \geq f_{ij}h \\ \dfrac{\phi^x_{Min} + \phi^y_{Min} + \left(2f_{ij}^2h^2 - (\phi^x_{Min} - \phi^y_{Min})^2\right)^{1/2}}{2} \; ; & |\phi^x_{Min} - \phi^y_{Min}| \leq f_{ij}h \end{cases}$$

Numeric Methods

In crowd Simulation there could be various ways to simulate the governing equations, and **I have chosen Finite Difference Approach to do so, in my project.**

Advection Equation

The Finite Difference scheme used for solving advection equation is **Lax Wendroff Scheme**

Given below is Lax Wendroff Scheme for Advection equation in one dimension (Where f is flux)

Lax Wendroff scheme:

$$f^{n+1/2}_{j+1/2} = \frac{1}{2}\left(f^n_{j+1} + f^n_j\right) - \frac{\Delta t}{2\Delta x}\left(F^n_{j+1} - F^n_j\right)$$

$$f^{n+1/2}_{j-1/2} = \frac{1}{2}\left(f^n_j + f^n_{j-1}\right) - \frac{\Delta t}{2\Delta x}\left(F^n_j - F^n_{j-1}\right)$$

$$f^{n+1}_j = f^n_j - \left(\frac{\Delta t}{\Delta x}\right)\left(F^{n+1/2}_{j+1/2} - F^{n+1/2}_{j-1/2}\right)$$

~~And~~ where, $F^n_j = f(f^n_j)\cdot f^n_j$

$= (1-f^n_j)\cdot f^n_j$

Eikonal Equation

The Finite Difference scheme used for solving advection equation is **Fast Sweep Scheme**

Given equation: $|\phi(x)| = f(x)$ on Ω

$\phi(x) = 0$ on Γ_{exit}

$\frac{\partial \phi}{\partial n} = 0$ on Γ_{wall}

$$\phi^x_{Min} = Min\left(\phi^{(n)}_{i-1,j}, \phi^{(n)}_{i+1,j}\right)$$

$$\phi^y_{Min} = Min\left(\phi^{(n)}_{i,j-1}, \phi^{(n)}_{i,j+1}\right)$$

$$\phi^{(n+1)}_{ij} = \begin{cases} Min\left(\phi^x_{min}, \phi^y_{Min}\right) + f_{ij}h ; & |\phi^x_{Min} - \phi^y_{Min}| \geq f_{ij}h \\ \dfrac{\phi^x_{Min} + \phi^y_{Min} + \left(2f_{ij}^2h^2 - (\phi^x_{min} - \phi^y_{Min})^2\right)^{1/2}}{2} ; & |\phi^x_{Min} - \phi^y_{Min}| \leq f_{ij}h \end{cases}$$

The Algorithm

Initialize the Density (Initial Condition)

Density at each point has to be specified at t=0

Now the density is known at each point at this time step

Speeds at every point is found

And this is done using Greenshields's method

$$f(\rho) = V_{Max}\left(1 - \frac{\rho}{\rho_{Max}}\right)$$

Potential at every point is obtained

Using Speed at each point and Fast Sweep algorithm, the potential at each point is obtained

$$f(\rho) = \frac{1}{\|\nabla\phi\|} = \frac{1}{\left[\left(\frac{\partial\phi}{\partial x}\right)^2 + \left(\frac{\partial\phi}{\partial y}\right)^2\right]^{1/2}}$$

$$\phi^x_{Min} = Min\left(\phi^{(n)}_{i-1,j}, \phi^{(n)}_{i+1,j}\right)$$

$$\phi^y_{Min} = Min\left(\phi^{(n)}_{i,j-1}, \phi^{(n)}_{i,j+1}\right)$$

$$\phi^{(n+1)}_{ij} = \begin{cases} Min(\phi^x_{Min}, \phi^y_{Min}) + f_{ij}h \ ; & |\phi^x_{Min} - \phi^y_{Min}| \geq f_{ij}h \\ \dfrac{\phi^x_{Min} + \phi^y_{Min} + \left(2f_{ij}^2h^2 - (\phi^x_{Min} - \phi^y_{Min})^2\right)^{1/2}}{2} \ ; & |\phi^x_{Min} - \phi^y_{Min}| \leq f_{ij}h \end{cases}$$

Velocity vector at every point is obtained

Using Speed at each point (Magnitude), and Potential Field (direction), velocity vector field is obtained

$$u = f(\rho)\,\frac{\left(-\frac{\partial\phi}{\partial x}\right)}{\|\nabla\phi\|} \qquad v = f(\rho)\,\frac{\left(-\frac{\partial\phi}{\partial y}\right)}{\|\nabla\phi\|}$$

Density at every point at next time step is obtained

Using the velocity at each time step, and density at the current time step the value of density at next time step is evaluated, using Lax Wendroff Scheme

$$\frac{\partial\rho}{\partial t} + \nabla\cdot(\rho\vec{u}) = 0$$

$$\frac{\partial\rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) = 0$$

Lax wendroff scheme:

$$\rho^{n+1/2}_{j+1/2} = \tfrac{1}{2}\left(\rho^n_{j+1} + \rho^n_j\right) - \frac{\Delta t}{2\Delta x}\left(F^n_{j+1} - F^n_j\right)$$

$$\rho^{n+1/2}_{j-1/2} = \tfrac{1}{2}\left(\rho^n_j + \rho^n_{j-1}\right) - \frac{\Delta t}{2\Delta x}\left(F^n_j - F^n_{j-1}\right)$$

$$\rho^{n+1}_j = \rho^n_j - \left(\frac{\Delta t}{\Delta x}\right)\left(F^{n+1/2}_{j+1/2} - F^{n+1/2}_{j-1/2}\right)$$

And where, $F^n_j = f(\rho^n_j)\cdot\rho^n_j$

$= (1-\rho^n_j)\cdot\rho^n_j$

Results

Results are presented as animations.